

# How Do Classical and Quantum Probabilities Differ?

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## **Abstract.**

I show that the classical coin tossing experiment involves two distinct definitions of probability, one ontological (the relative frequencies of initial deterministic states) and another empirical (the relative frequencies of observations). In quantum theory, I argue, only the latter definition can be invoked, since a single superposition state can give rise to multiple observation experiences. This difference explains why the present statistical quantum mechanics is an ontological dead end, despite its enormous pragmatic success. To get at the ontological content of quantum theory, I propose that the observations themselves can be interpreted on a different footing, without reference to determinate detector states.

**Keywords:** quantum probabilities, measurement problem, naïve realism, quantum ontology, relational properties, macroscopic realism, macroscopic quantum mechanics, qualia

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## **INTRODUCTION**

Let us begin by analyzing the notion of probability in a classical coin tossing experiment, and contrasting it with the notion of probability in quantum theory (QT). In classical coin tossing, there are two chains of events: the sequence of classically determinate states that obtain in the world prior to and independent of our acts of observation, and the sequence of observed outcomes they entail. The former is a theoretical construct; the latter is empirically given. The term ‘probability’ can refer to either. In contrast, QT can describe only the relative frequencies of observation events since the same superposed state underlies each trial. This limitation makes QT irreducibly statistical, and an ontological dead end.

To move toward an ontological interpretation, one in which observations would be linked to states that exist prior to and independent of observations, I propose that an observation experience, usually reported using *ordinary language* as a state of a macroscopic measuring apparatus (“the top detector clicked”), can instead be reported as our direct experience (“a sound was heard”). The first statement pertains to a localized, space-time event in the external world: a position measurement. But it also entails presupposing naïve realism — the viewpoint that macroscopic measuring devices have determinate states independent of our experience— in order to go toward a realistic

interpretation. As the famous cat paradox indicates, naïve realism is suspect even at the macroscopic level in QT. It also leads to decidedly profound problems at the microscopic level. An alternative is to treat observations as pure sense experiences, and move on to a different version of macroscopic realism: matter as it gives rise to the *qualitative* aspects of phenomenal objects. It might enable us, as I intend to argue, to relate quantum physical reality to observations in a manner more befitting the non-classical quantum formalism.

## CLASSICAL COIN TOSSING AND ELEMENTARY PROBABILITY THEORY (EPT)

Let H (heads) and T (tails) denote the two possible outcomes when we toss a coin. The toss results form a “random” sequence Hs and Ts in a limited sense: no matter how often we toss the coin, the observed sequence is not sufficient to predict the *next* toss result. Nevertheless, there *is* an order governing the ensemble of toss results: the observed frequencies of Hs and Ts in a long trial approach  $\frac{1}{2}$ .

To account for this order, a measure called “probability” ( $p$ ) is introduced for each distinct outcome, with the condition,  $\sum_{i=1}^2 p(i) = 1$ . Assuming that nature does not favor either outcome,  $p(1)=p(2)=\frac{1}{2}$  in a given toss, accounting for the *limit* approached by observed relative frequencies in a series of tosses.

Since the probability is defined in relation to possible outcomes in a *single trial*, we cannot *predict* which particular outcome will appear in any given toss. However, we are able to assert that *one* outcome has appeared in a given toss by (and only by) *looking* at the tossed coin.

To avoid the conclusion that the result is not a real event until we look at it, we simply assume that the motion of the tossed coin obeys classical mechanics. Each toss result is then entirely determined by the coin’s initial classical mechanical state. This classical physical state would be uniquely distinct for each toss. However, we can categorize them into just two generic states corresponding to the two possible observations: heads-producing ( $S_H$ ) or tails-producing ( $S_T$ ) states. Thus, given a sequence of 7 toss results such as HTTHHHT, we can presume that the initial states causing the observed results occurred in nature in the sequence:  $S_H, S_T, S_T, S_H, S_H, S_H, S_T$ . We invoke probabilities only because this initial state, which occurs in nature in each trial, is too complicated for us to calculate.

Let us call this physics-based reasoning “elementary probability theory” (EPT). It invokes both mathematical and physical reasoning, as described above. The mathematical part defines the space  $\Omega=\{r_1, r_2, \dots\}$  of distinct observation outcomes ( $r_i$ ’s) and assigns a probability measure to each. The physical part is the idea that each  $r_i$  has a corresponding physical state of the coin  $S_i$  that obtains in the world prior to and independent of our act of observation. Thus, when we speak of the probability for outcome  $r_i$  (H or T) in a coin toss, we are *primarily* speaking of the probability for the physical state  $S_i$  ( $S_H$  or  $S_T$ ) to occur; in terms of their relative frequency of occurrence in nature.

In QT, however, a single superposed state underlies all observation outcomes. Nevertheless,  $\int_x^{x+dx} \psi\psi^* dx$  is introduced as the probability of *finding* a particle in the

specified interval (Born's rule, 1-dimensional case). This statement really involves three stages of reasoning:

A:  $P(x) = \int_x^{x+dx} \psi \psi^* dx$  gives the probability that *we* will have an observation experience in the specified spatial interval in our private phenomenal space.

B:  $P(x)$  gives the probability that a detector placed in that interval will fire.

C:  $P(x)$  gives the probability of *finding* a particle in the specified spatial interval in the 'real' world.

Textbooks do not even mention stages A & B. Uncritically embracing B & C has obliged physicists to introduce the projection postulate, which in turn leads to the famed and to-date unsolved "measurement problem"— explaining when and how the transition from a superposed state to an eigenstate occurs. By limiting the content of Born's rule to A, we could sidestep these problems. In QT, there is clear justification to jettison C since *one single* superposed state underlies all observation outcomes. Thus, the term probability can only refer to A. The talk of probability of eigenstates can have no realist content since the eigenstates *co-appear with the observations*.

How can we give up B? It *seems* that detector clicks are unavoidably "given" in our experience. In the next section, I shall advance a key argument that at least in principle, it is possible to interpret our sense impressions using ordinary language without reference to detector states. Such re-interpreted observations can potentially be linked directly to the underlying superposed state. That would help us avoid the measurement problem and get at a truly realist interpretation of QT.

## **EXPERIENCE AND REALITY IN ORDINARY LANGUAGE DESCRIPTIONS**

Observations are, first and foremost, our sense experiences that must be described using ordinary language (OL) in order to have significance and be communicable, as Bohr emphasized. However, unlike Bohr who needlessly decreed that "all new experience makes its appearance within the frame of our customary points of view and forms of perception" ([1], p. 1), I shall explore the possibility for describing our sense impressions in a non-customary manner, using an alternative range of ordinary language notions.

The customary point of view is that the macroscopic objects we cognize are assumed to exist 'out there' independent of our experience. Physicists too have traditionally *assumed* this naïve realism at the level of observations. It has provided a natural starting point for physicists to visualize phenomena in space and time and then develop mathematical descriptions of their dynamics. In QT, however, the development of the mathematical formalism *preceded* space-time visualization of the phenomena. Planck directly fitted a mathematical formula to the observed data, and *then* tried to find a corresponding mechanical picture. That approach became a tradition in QT, up to the time when Schrödinger derived his wave equation. Now, naïve realism *at the level of observations* no longer suffices to yield a consistent space-time interpretation of quantum dynamics.

Under the circumstances, it is reasonable to look for an *alternative conception of macroscopic realism*. Naïve realism is so ingrained that it may seem difficult to entertain an equally pragmatic alternative, much less work out a new application of QT based on it. However, we do invoke different versions of macroscopic realism in our everyday thinking. For example, the statement “the pen is inside my pocket” refers to the absolute location of a ‘real’ pen in the world. On the other hand, the statement “the moon is inside the window” refers to a position of the *phenomenal moon* of our experience, which is a *relation* between the moon, the window, and an observer, all real in the world. In other words, even a ‘real’ macroscopic object can have a position “only when we see it”, in the phenomenal world.

To allow for two everyday conceptions of ‘position’ and the two differing underlying conceptions of reality they relate to, I have introduced the idea that ordinary language functions in two modes: A *P-mode*, which describes our observations *qua* outer experiences; and an *R-mode*, which describes the underlying physical state of affairs [2]. Upon this terminology, macroscopic realism is the thesis that for every P-mode description, a corresponding R-mode statement exists in OL. Naïve realism, which is but one version of macroscopic realism, assumes that the *same* OL statement in P-mode also serves as an R-mode statement to describe reality. According to naïve realism, the same OL statement “the top detector clicked” can describe our experience (P-mode) or the underlying real world (R-mode). This is the current stance of physics.

The P- and R-mode distinction makes it possible to abandon naïve realism in QM without having to abandon macroscopic realism itself. We simply look for *an alternative R-mode* description in OL to interpret the observations.

To make further progress, I now formulate the classical theory of measurement using the above terminology, as having the following steps:

1. An observation is a P-mode experience.
2. This observation can be interpreted in classical R-mode involving naïve realism.
3. Thus, an observation refers to a change in the state of a real macroscopic measuring device M in the world
4. The state of M yields a *value* (measurement result), which is the value of property *p* of an object O under measurement.
5. The property *p* had this value prior to and independent of our observation.

Clearly, all of the steps above cannot be maintained in QT. Which steps have to be avoided in order to arrive at a full quantum theory of measurement?

The standard or “Copenhagen” interpretation holds that in QT only step 5 needs to be jettisoned. Quantum mechanical observables are treated as contextual properties: they have determinate values only in the context of an actual measurement. This precludes the possibility of an ontological interpretation.

Bohr’s own interpretation was distinctly different [3]. He held that steps 3 and 4 also fail in QT because “an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation” ([1], p. 64). Thus, Bohr’s approach too precluded an ontological interpretation.

We can go past both interpretations by jettisoning step 2 also, which is already known to be suspect due to the cat paradox. The idea that the detector has a classically

determinate state prior to and independent of measurement is as incompatible with QT as the idea the cat was dead or alive prior to and independent of measurement. Hence, our best chances for quantum realism lies in not letting our description of observations refer to the state of the measuring device at all. The statement “a sound was heard”, taken as a P-mode statement, just describes our observation experiences, and entails no necessary reference to the detector states. By restricting ourselves to such statements, we can avoid invoking steps 2-4 of classical measurement theory.

However, to clinch quantum realism from this starting point, we have to objectify the *sound experience* properly. The classical physics idea of sound as compressions and rarefactions that travelled to the ear will not do. We need a new R-mode interpretation, to find which I am proposing *we change our very notion of scientific realism*. Instead of the classical dogma articulated by Einstein —“the programmatic aim of physics is to describe a physical reality as it is *thought* to exist independent of our acts of observations upon them”, I propose that in QT, we view the task of physics as describing a physical reality that underlies and gives rise to our observations qua sense experiences.

Observations interpreted using classical R-mode involving determinate detector states cannot be directly linked to the underlying superposed state, leaving QT irreducibly statistical. By employing new P-mode observations such as “I heard a sound” (and likewise for other sense modalities), and looking for an alternative R-mode that bypasses reference to detector states, we can hope to link the observations directly to superposed states, and develop a quantum theory of qualia.

To put it another way, there are *two* conceptions of macroscopic objects: the phenomenal object of our experience, and the ‘real’ macroscopic object presumed to exist in the external world. So far, in QT, we have related our observations to ‘real’ (i.e., classical) objects that are described by primary properties. My proposal is to let the quantum superposed state refer to the physical reality underlying and giving rise to the phenomenal objects which are best described in terms of their qualitative properties. In this regard, I have discussed elsewhere a notion of relational properties that are in-between primary and secondary properties, and which can describe a macroscopic object as a semantic symbol rather than a classical physical token [4], [5].

## CONCLUSION

In classical physics, we objectify the observation experiences as events in a real world and then let our physics describe this world. Currently, QT continues this approach. The present paper suggests where we can begin to look to avoid the intermediate step of classically objectifying the observations, and interpret QT as a theory directly linking the qualitative properties of macroscopic objects to underlying superposed states. This requires developing a new R-mode of OL that is only alluded to in this paper. Perhaps  $\hbar$  gives the room for the mathematical formalism to be interpreted using either approach. The idea of two different P-mode descriptions of observations (and correspondingly two different R-mode interpretations of underlying reality) may introduce a complementarity deeper than the wave and particle descriptions currently adopted in QT. These are topics for future papers.

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